

# Probing Quark Distribution Amplitudes Through Generalized Parton Distributions at Large Momentum Transfer

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(Dated: February 1, 2008)

## Abstract

In the large momentum transfer limit, generalized parton distributions can be calculated through a QCD factorization theorem which involves perturbatively-calculable hard kernels and light-cone parton distribution amplitudes of hadrons. We illustrate this through the  $H_q(x, \xi, t)$  distribution for the pion and proton, presenting the hard kernels at leading order. As a result, experimental data on the generalized parton distributions in this regime can be used to determine the functional form of the parton distribution amplitudes which has thus far been quite challenging to obtain. Our result can also be used as a constraint in phenomenological GPD parametrizations.

arXiv:hep-ph/0309085v2 17 Sep 2003

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It has been established in perturbative quantum chromodynamics (pQCD) that hard exclusive processes in the asymptotic limit depend on the non-perturbative light-cone parton distribution amplitudes of hadrons [1, 2, 3, 4]. However, the functional form of these amplitudes in parton momenta  $x_i$  has been very difficult to determine either from experimental data or from theoretical calculations [5, 6]. For instance, the asymptotic electromagnetic form factors depend only on an integral of the distribution amplitudes [1, 3]. In the pion-nucleus diffractive production of two jets, one can in principle learn about the shape of the quark distribution amplitude in the pion, but the process is not infrared safe as has been pointed out recently [7]. Poor knowledge on the distribution amplitude in the proton has been the main obstacle in deciding at what momentum transfer the asymptotic pQCD calculation is relevant [8, 9, 10].

In this paper, we study the generalized parton distributions (GPDs) [11, 12] of hadrons in the large momentum transfer limit. The GPDs are a new class of hadron observables which combine the physics of electromagnetic form factors and Feynman parton distributions, and are related to quantum phase-space distributions of the partons through Fourier transformation [13]. Apart from the renormalization scale, they depend on the momentum transfer  $t$  as in a form factor, light-cone momentum  $x$  as in a parton distribution, and the projection of the momentum transfer along the light-cone direction  $\xi$ , also known as the skewness parameter.

We report here that the GPDs in  $-t \rightarrow \infty$  limit are calculable through QCD factorization in which the non-perturbative physics is included in the light-cone distribution amplitudes of hadrons. Using this, the functional form of the distribution amplitudes can be studied through the GPDs' dependence on  $x$  and  $\xi$ . Conversely, our result provide a constraint on phenomenological GPD parametrizations. The GPDs at large- $t$  can be measured, for example, from deeply-virtual Compton scattering or hard exclusive meson production or doubly-virtual Compton scattering in the kinematic regime  $Q^2 \gg -t \gg \Lambda_{\text{QCD}}^2$  in which the factorization theorems for scattering amplitudes have been proven [14]. However, it can be experimentally challenging to measure the cross sections in this regime because of additional power suppression in  $t$ ; we will not explore this issue here.

We illustrate our main point first by considering the generalized parton distribution  $H(x, \xi, t)$  for the pion, defined through

$$H_q(x, \xi, t) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle \pi; P' \left| \bar{\psi}_q \left( -\frac{\lambda}{2} n \right) \not{n} \right. \right. \\ \left. \left. \times \mathcal{P} e^{-ig \int_{\lambda/2}^{-\lambda/2} d\alpha n \cdot A(\alpha n)} \psi_q \left( \frac{\lambda}{2} n \right) \right| \pi; P \right\rangle, \quad (1)$$

where  $P$  and  $P'$  are the initial and final state pion momenta, respectively,  $t = (P - P')^2$ , and  $\mathcal{P}$  indicates path-ordering for the light-cone gauge link. Introducing  $\bar{P} = (P + P')/2$  along the  $z$  direction and the conjugation light-cone four-vector  $n$ , such that  $n^2 = 0$  and  $n \cdot \bar{P} = 1$ , the skewness parameter is the projection of the momentum transfer  $P' - P$  along  $\bar{P}$  direction,  $\xi = -n \cdot (P' - P)/2$ . The initial and final light-cone momenta of the quarks are then  $n \cdot k = x + \xi$  and  $n \cdot k' = x - \xi$ , respectively.

In the large momentum transfer limit, one can calculate the above GPD using the pQCD factorization formalism which has been widely applied to electromagnetic and other form factors [1, 2, 3, 15]. The leading pQCD contribution is shown in Fig. 1, where the initial and final pion states are replaced by the light-cone Fock component with the minimal number of partons. The circled crosses in the diagrams represent the bilocal quark operator in Eq. (1).

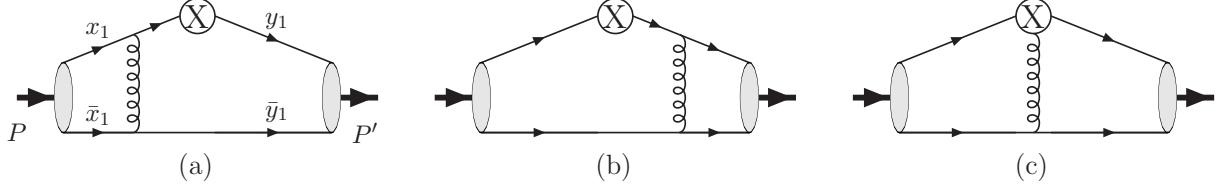


FIG. 1: *Leading pQCD diagrams contributing to the pion's generalized parton distribution  $H(x, \xi, t)$  at large  $-t$ . The circled crosses represent the non-local quark operator.*

The hard part responsible for the large momentum transfer contains a single gluon exchange just like in the electromagnetic form factor. In the first two diagrams (a) and (b), there is a hard gluon exchange between the two quark lines, and in the third one, there is a gluon coming from the gauge link. Since the transverse momenta of the quarks are expected to be on the order of  $\Lambda_{\text{QCD}}$ , we may ignore them in calculating the hard part. Thus we can effectively integrate out  $k_\perp$  in the pion wave function to obtain the distribution amplitude,  $\phi(x) = \int \frac{d^2 k}{(2\pi)^3} \psi(x, k_\perp)$ . The parton transverse momenta flowing into the hard part are now taken to be zero.

The result of the above pQCD analysis is a factorization formula for the GPD at large  $t$  in terms of the quark distribution amplitude

$$H_q(x, \xi, t, \mu) = \int dx_1 dy_1 \phi^*(y_1, \mu) \phi(x_1, \mu) T_{Hq}(x_1, y_1, x, \xi, t, \mu), \quad (2)$$

where  $T_{Hq}$  is the hard part and can be calculated as a perturbation series in  $\alpha_s$ . All quantities in the above equation depend on the renormalization scale  $\mu$ . The  $\mu$ -dependence in the hard part must be such that it accounts for the difference between the GPD and the distribution amplitude.

The leading contribution to the hard part can be calculated straightforwardly,

$$T_u(x, x_1, y_1) = \frac{4\pi\alpha_s C_F}{\bar{x}_1 \bar{y}_1 (-t)} \delta(x - \lambda_1) \left[ (1 - \xi) + \frac{1 - \xi^2}{\lambda_1 - \tilde{\lambda}_1} \right] + \text{h.c.}, \quad (3)$$

where  $C_F = 4/3$ , h.c. stands for a term obtained by exchange  $x_i$  and  $y_i$ , and  $\xi$  and  $-\xi$ ,  $\lambda_1 = y_1 + \bar{y}_1 \xi$ ,  $\tilde{\lambda}_1 = x_1 - \bar{x}_1 \xi$  ( $\bar{x} = 1 - x$ ). Since  $0 < y_1 < 1$ , the first term contributes when  $x > \xi$ ; whereas the second term contributes when  $x > -\xi$ , which indicates an up-anti-up pair contribution. The anti-quark is generated through the one-gluon exchange on the top of the valence wave function. The GPD for the down quark  $H_d(x, \xi, t)$  can be obtained from that of the up quark through simple charge symmetry,  $H_d(x, \xi, t) = -H_u(-x, \xi, t)$ .

The above result can be translated into one for the moments of the GPDs  $H_q^{(n)}(\xi, t) = \int_{-1}^1 dx x^{n-1} H_q(x, \xi, t)$ . In fact, the factorization formula applies for the individual moments,  $H_q^{(n)}(\xi, t) = \int dx_1 dy_1 \phi^*(y_1) \phi(x_1) T_{Hq}^{(n)}(x_1, y_1, \xi, t)$ , where  $T_{Hq}^{(n)}(x_1, y_1, \xi, t)$  is simply the  $n$ -th moment of  $T_{Hq}(x_1, y_1, x, \xi, t)$ . For the up quark in the pion, we have,

$$T_u^{(n)}(\xi, t) = \frac{4\pi\alpha_s C_F}{\bar{x}_1 \bar{y}_1 (-t)} \left[ (1 - \xi) \lambda_1^{n-1} + (1 + \xi) \tilde{\lambda}_1^{n-1} + (1 - \xi^2) \sum_{m=0}^{n-2} \lambda_1^m \tilde{\lambda}_1^{n-m-2} \right], \quad (4)$$

which contains both even and odd powers of  $\xi$ . For  $n = 1$ , the above reproduces the hard part in the QCD factorization formula for the pion form factor [1, 2, 3]. For  $n = 2$ ,

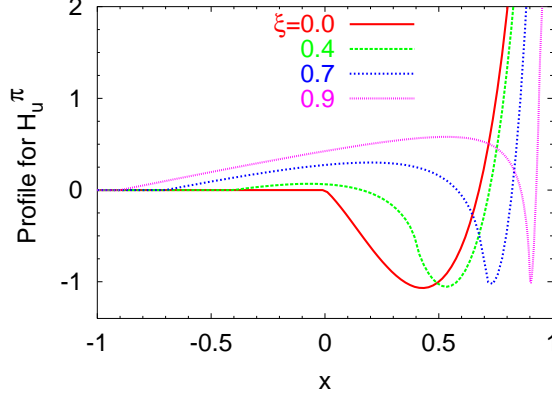


FIG. 2: The profile function of  $H(x, \xi, t)$  for the pion in the asymptotic limit.

$T_u^{(2)}(\xi, t) = 4\pi\alpha_s C_F / (\bar{x}_1 \bar{y}_1 t) [(x_1 + y_1 + 1) + 2(x_1 - y_1)\xi + (x_1 + y_1 - 3)\xi^2]$ . It is easy to see that the linear-dependence in  $\xi$  does not contribute to  $H_u^{(1)}(\xi, t)$  because of the symmetry in the initial and final states. For the same reason, all odd powers of  $\xi$  in  $T_u^{(n)}$  do not contribute to the GPD moments.

It has been suggested from the dijet production [7] and  $\gamma^*\gamma \rightarrow \pi$  transition [16] that the pion distribution amplitude at  $\mu > 2$  GeV is very close to the asymptotic amplitude  $\sqrt{6}f_\pi x(1-x)$  [1, 2, 3]. If so, we can make the prediction for the  $H_u$  as follows,

$$H_u(x, \xi, t) = \frac{16\pi\alpha_s f_\pi^2}{(-t)} \left\{ \theta(x - \xi) \frac{(x - \xi)}{(1 - \xi)} \times \left[ -1 - \frac{(x + \xi)}{(1 + \xi)} \log \frac{(1 - x)^2}{(x + \xi)^2} \right] + (\xi \rightarrow -\xi) \right\}, \quad (5)$$

which is continuous at  $x = \xi$  and  $x = -\xi$ . The quantity in the braces is a profile function and is plotted for four different  $\xi$  in Fig. 2. The function diverges at  $x = 1$ , indicating the breaking down of  $1/t$  expansion. This divergence generates a slow decrease of the GPD moments at large  $n$ , and is present even when  $\xi = 0$ . We note that the limit  $x \rightarrow 1$  and  $-t \rightarrow \infty$  may not be interchangeable. If we take the limit  $x \rightarrow 1$  first,  $H_q$  may not vanish in the subsequent  $-t \rightarrow \infty$  limit because the pion momentum is now carried by a single quark.

Now we turn to the proton case. The factorization formula for the GPD  $H_q$  takes a similar form

$$H_q(x, \xi, t) = \int [dx][dy] \Phi_3^*(y_1, y_2, y_3) \Phi_3(x_1, x_2, x_3) T_{H_q}(x_i, y_i, x, \xi, t), \quad (6)$$

where  $[dx] = dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)$ , and  $\Phi_3(x_i)$  is the three-quark distribution amplitude [17].

In the leading order in  $\alpha_s$ , there are three classes of diagrams contributing to the hard part, each with a representative shown in Fig. 2. The first class consists of diagrams with two-gluon exchanges not attached to the non-local operators. Shown in Fig. 2.1 is one of the 14 possible diagrams in this class. The second class has one-gluon coming from the gauge link in the non-local operator, and the third class with two gluons. To calculate the hard part, we arrange the first quark to have spin up, the second spin down, and the third spin

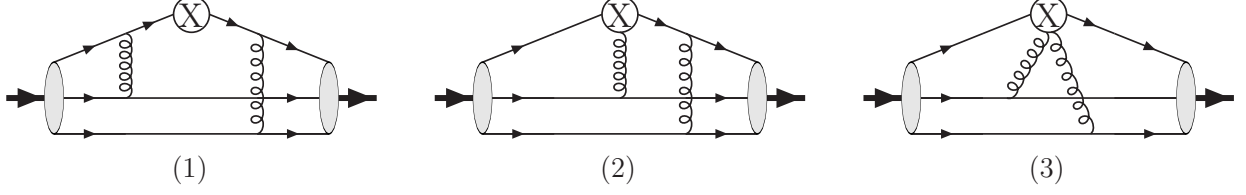


FIG. 3: Representatives from three classes of QCD diagrams contributing to the proton GPD  $H_q(x, \xi, t)$  in the asymptotic limit.

up again, with Feynman momentum  $x_1, x_2, x_3$  for the incoming quarks, and  $y_1, y_2$ , and  $y_3$  for outgoing quarks, respectively. We use  $T_i$  to denote the hard part with the non-local operator inserted on the line  $i$ . Then the hard part from the proton is

$$\begin{aligned} T_u^p &= \frac{1}{3}(2T_1 + T_2 + T_3 + T'_1 + T'_3) \\ T_d^p &= \frac{1}{3}(T_2 + T_3 + T'_2) \end{aligned} \quad (7)$$

$T'_i$  is obtained from  $T_i$  by interchanging  $y_1$  and  $y_3$ .

Our result for the hard part is

$$\begin{aligned} T_1 &= \frac{2\pi^2 C_B^2 \alpha_s^2}{t^2} \left\{ K_{11} \delta(x - \lambda_1) + K_{12} \frac{\delta(x - \lambda_1) - \delta(x - \tilde{\lambda}_1)}{\lambda_1 - \tilde{\lambda}_1} \right. \\ &\quad \left. + K_{13} \frac{\delta(x - \lambda_1) - \delta(x - \eta_1)}{\lambda_1 - \eta_1} + K_{14} \frac{\delta(x - \lambda_1) - \delta(x - \eta_1)}{(\lambda_1 - \eta_1)(\eta_1 - \tilde{\lambda}_1)} \right\} + \text{h.c.} \end{aligned} \quad (8)$$

$$\begin{aligned} T_2 &= \frac{2\pi^2 C_B^2 \alpha_s^2}{t^2} \left\{ K_{21} \delta(x - \eta_2) + \left[ K'_{13} \frac{\delta(x - \lambda_2) - \delta(x - \eta_2)}{\lambda_2 - \eta_2} + (1 \leftrightarrow 3) \right] \right. \\ &\quad \left. + K'_{14} \frac{\delta(x - \lambda_2) - \delta(x - \eta_2)}{(\lambda_2 - \eta_2)(\eta_2 - \tilde{\lambda}_2)} \right\} + \text{h.c.}, \end{aligned} \quad (9)$$

where  $C_B = 2/3$ ,  $\lambda_i = y_i + \bar{y}_i \xi$ ,  $\eta_1 = 1 - x_3 - y_2 + (y_2 - x_3)\xi$ , and  $\eta_2 = \eta_1(1 \leftrightarrow 2)$ . The functions  $K_{ij}$  are defined as

$$\begin{aligned} K_{11} &= \frac{1}{x_3 y_3 \bar{x}_1 \bar{y}_1} + \frac{1}{x_2 y_2 \bar{x}_1 \bar{y}_1} - \frac{1}{x_2 y_2 x_3 y_3 \bar{x}_3 \bar{y}_1} \\ K_{12} &= \frac{1 - \xi}{x_2 y_2 \bar{x}_1 \bar{y}_1} + \frac{1 - \xi}{x_3 y_3 \bar{x}_1 \bar{y}_1} \\ K_{13} &= \frac{1 - \xi}{x_2 y_2 x_3 y_3 \bar{x}_3} \\ K_{14} &= \frac{(1 + \xi)(1 - \xi)}{x_2 y_2 x_3 y_3} \\ K_{21} &= \frac{1}{x_1 y_1 x_3 y_3 \bar{x}_3 \bar{y}_1}, \end{aligned} \quad (10)$$

and  $K'_{ij} = K_{ij}(1 \leftrightarrow 2)$ .  $T_3$  can be obtained from  $T_1$  by exchanging  $(x_1, y_1)$  and  $(x_3, y_3)$ , respectively. From the above, we can calculate the GPD moments for the nucleon in a

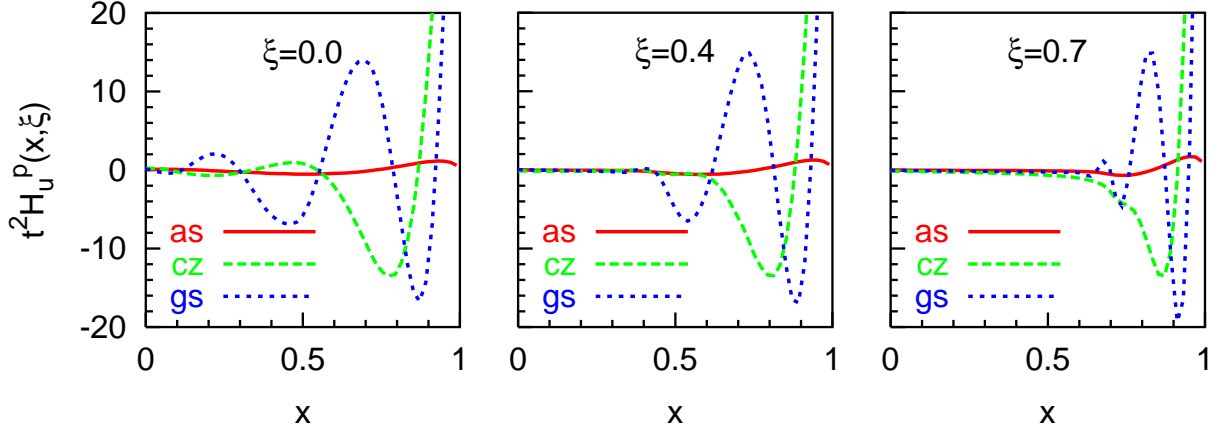


FIG. 4:  $t^2 H_u(x, \xi, t)$  for the proton at  $-t = 20 \text{ GeV}^2$ . CZ refers to the Chernyak and Zhitnitsky amplitude, GS the Gari and Stefanis amplitude, and AS the asymptotic amplitude.

factorization form. If we take the first moment, we recover the pQCD prediction for the Dirac form factor  $F_1(Q^2)$  [1, 2, 3]. If we take the second moment, we find the pQCD prediction for gravitational form factors  $A(Q^2)$  and  $C(Q^2)$  at large  $Q^2$  [11]. The contribution to  $C(Q^2)$  is zero at this order in  $1/t$ . This is because  $C(Q^2)$  also contribute to the helicity-flip GPD  $E(x, \xi, t)$  which is sub-leading in the large  $-t$  limit.

One can make a numerical calculation of  $H_u(x, \xi, t)$  using various model amplitudes in the literature [3, 5, 10]. Using the strategy of Ref. [18], we have computed  $t^2 H_u$ , shown in Fig. 4, for 3 different  $\xi$  at  $t^2 = -20 \text{ GeV}^2$  with the asymptotic, Chernyak-Zhitnitsky, and Gari-Stefanis amplitudes. Although the CZ and GS amplitudes both give reasonable account of data on  $F_1^p$  for  $-t \geq 10 \text{ GeV}^2$ , the two yield very different predictions for the GPD. Note that the scale of  $H_u$  is strikingly large; a relatively small Dirac  $F_1$  is resulted from the cancellation in the integration.

In summary, we have obtained a QCD factorization formula for the generalized parton distributions in terms of the non-perturbative light-cone distribution amplitudes and perturbatively-calculable hard kernels. We have calculated the hard kernels for the pion and the nucleon in the leading-order in  $\alpha_s$ . As a result, data on the GPDs in the large- $t$  regime provides a way to constrain the functional form of the distribution amplitudes.

Note added: After this paper was finished, we learned that the pion case has been studied in Ref. [19], our result differs from that in the paper. Studying GPD in the large- $t$  limit was first done in [20] for  $\gamma^* \gamma \rightarrow \pi\pi$ . We thank Diehl for pointing this out to us.

We are grateful for useful conversations with Stan Brodsky, Jian-Ping Ma, and Dieter Müller. This work was supported by the U. S. Department of Energy via grant DE-FG02-93ER-40762.

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